Technical Bulletin

Modeling Maximum Core Utilization

One of the main goals of optimized magnetics design is to select a core that will be used to the fullest. This goal is defined as *utilization*,

$$U = \frac{\text{maximum amount used}}{\text{maximum amount the device is capable of}}$$

Ideally, $U = 1$ for magnetic cores, and when U is maximized, core volume is minimized. To maximize U, what exactly is being utilized? The function of a core is to transfer (for transformers) and also store (for coupled inductors) magnetic energy. This maximizes the flow of *transfer power* through the magnetic component. Consequently, the goal is to find the conditions under which maximum energy is stored, or to maximize energy density in the core.

Core utilization is maximized by maximizing the two field quantities that determine transfer energy: saturation-limited average field intensity, $\overline{H}$, at the core operating point (op-pt), and the B-field ripple (~), $B_\sim = \Delta B$, limited by allowable core power loss density, $\overline{p}_c$, and frequency, f_s. (Frequency can also be optimized for a given core material; see the bulletin, "Mathematics of Frequency Optimization for Maximum Power Transfer.") $\overline{H}$ drives the core to the desired extent toward saturation, and $B_\sim$ amplitude (^), $\hat{B}_\sim = \Delta B/2$ – the horizontal axis on core-material power-loss graphs – drives the core to $\overline{p}_c$, the maximum allowable power loss density in the core for a given frequency and core material.

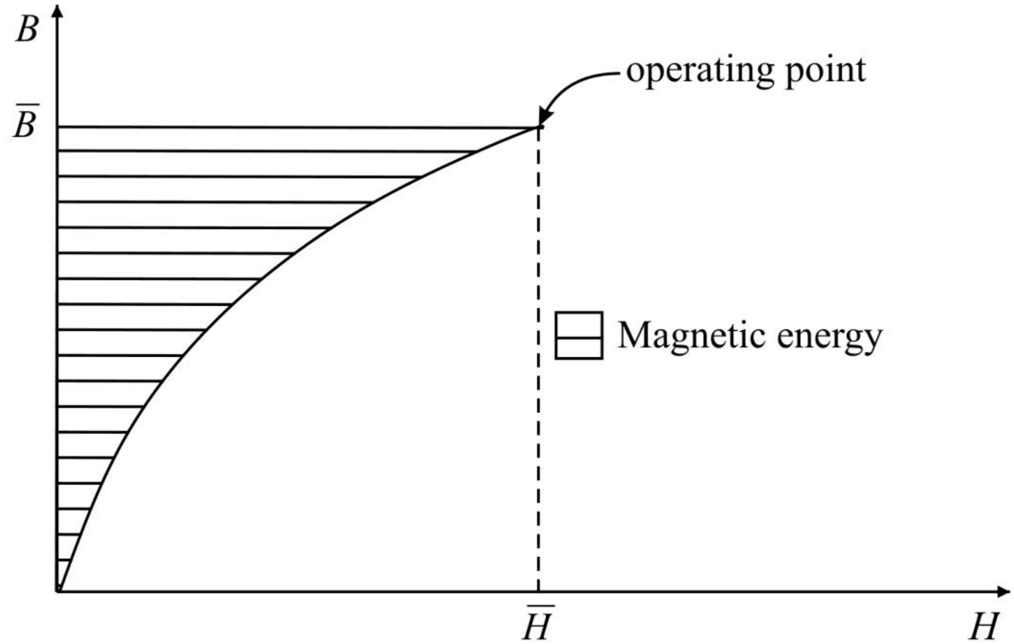
By dissipating the maximum acceptable magnetic power loss, the core is not oversized, nor is it if driven to an acceptable limit of saturation. Both of these maxima can be achieved simultaneously, resulting in either maximum transfer energy for a given volume or smallest core volume for a given transfer energy.

The core energy density, referred to the field, is

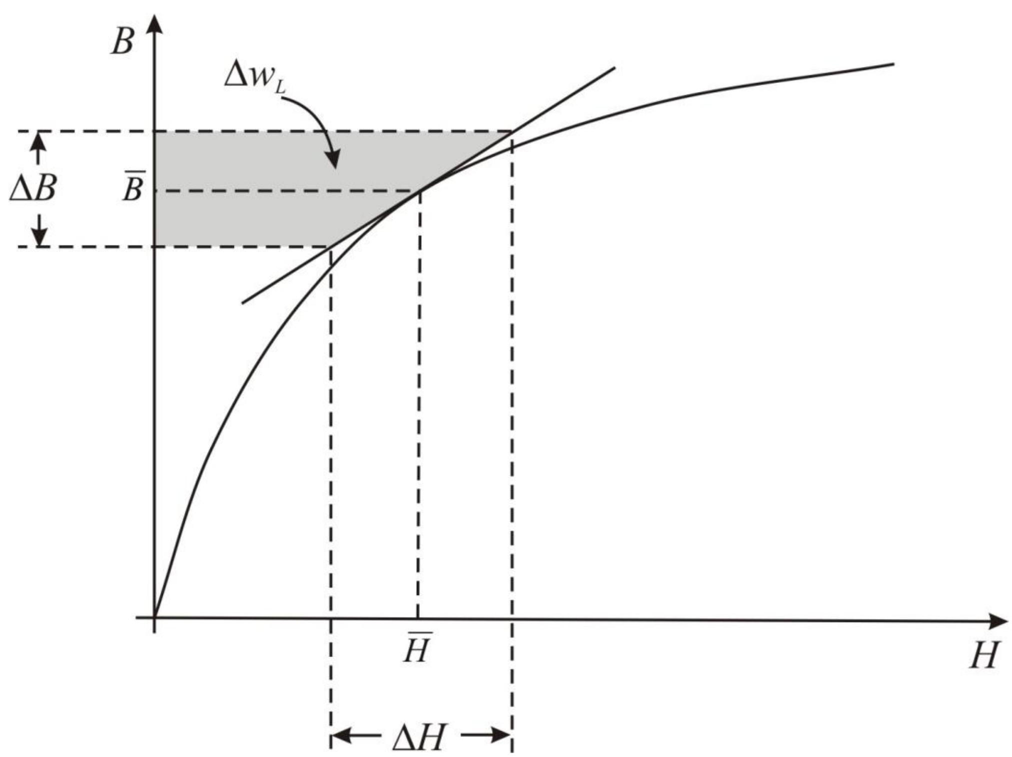
$$w_L = \frac{W_L}{V} = \int H \cdot dB = \int \left(\frac{B}{\mu}\right) \cdot dB = \frac{B^2}{2 \cdot \mu}, \text{ constant } \mu \text{ (linear)}$$

and is depicted below on a $B(H)$ graph. For linear magnetics, $B(H)$ is linear and μ, the slope of the $B(H)$ function, is constant; then incremental μ and total-variable μ are equal, and the total linear energy stored in a core of volume, V is

$$W_L = \frac{B^2 \cdot V}{2 \cdot \mu}$$



Powder core, ferrite, and tape wound core magnetics have a nonlinear *B*(*H*) but (like transistors) can be linearized around an op-pt $(\overline{H}, \overline{B})$, as shown below, with excursions of $\hat{B}_{\sim} = \Delta B / 2$ each side of $\overline{B}$ that are relatively small or *incremental*.



The peak-to-peak variation around $\overline{B}$ can be expressed as

$$\Delta B = \left(\overline{B} + \frac{\Delta B}{2}\right) - \left(\overline{B} - \frac{\Delta B}{2}\right)$$

To find $\Delta W_L = \dfrac{\Delta B^2 \cdot V}{2 \cdot \mu}$ we need

$$\Delta B^2 \equiv \Delta(B^2) = \left(\overline{B} + \frac{\Delta B}{2}\right)^2 - \left(\overline{B} - \frac{\Delta B}{2}\right)^2 = 2 \cdot \Delta B \cdot \overline{B}$$

The incremental per-cycle energy transferred is thus

$$\Delta W_L = \frac{V}{2 \cdot \mu} \cdot (\Delta B^2) = \frac{V}{\mu} \cdot \Delta B \cdot \overline{B} = \frac{V}{\mu} \cdot \Delta B \cdot (\mu \cdot \overline{H})$$

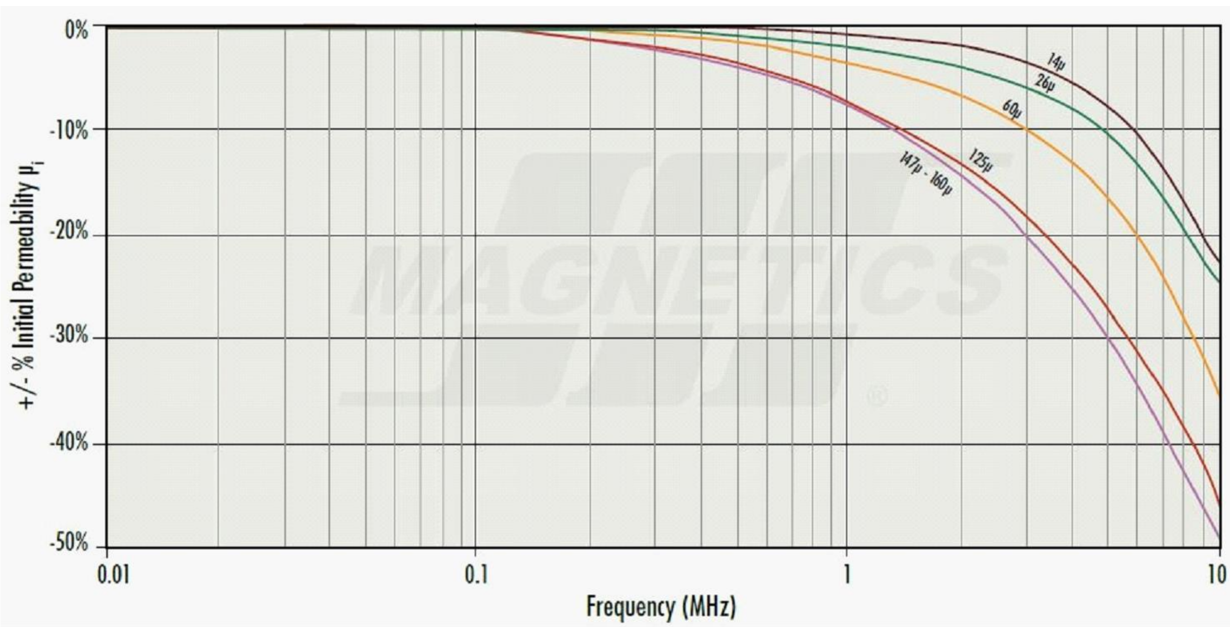
where μ is the static μ because it relates the static B and H of the op-pt: $\overline{B} = \mu_{static} \cdot \overline{H}$. The incremental μ is used instead with incremental variables: $\hat{B}_{\sim} = \mu \cdot \hat{H}_{\sim}$. Simplifying the algebra results in the basic incremental energy transfer formula as expressed from a field standpoint (in field variables):

$$\Delta W_L = [\Delta B \cdot \overline{H}] \cdot V$$

Maximum transfer energy is achieved when the core is driven with as large of a ΔB as the thermal limit allows and with as large of an $\overline{H}$ as saturation allows. Then the rate at which this core energy is transferred from input to output is the average transfer power, proportional to f_s.

Core Comparison

To give these equations a concrete context, we compare two Magnetics toroid powder cores with the same core geometry (volume, $V = 960\ \text{mm}^3$), and relative permeability of $\mu_r = 60$ (60μ), a High Flux 58381 and an Edge 59380, both operating at 100 kHz. To check the effect of frequency on μ, the $\mu(f)$ graph shows that at 100 kHz (0.1 MHz), μ shows no significant decrease from the low-frequency value for 60μ material. The -10% roll off value (f_μ) is 3.0 MHz for 60μ High Flux.



The core data is given below. The geometry and size-dependent magnetic parameters are essentially equal.

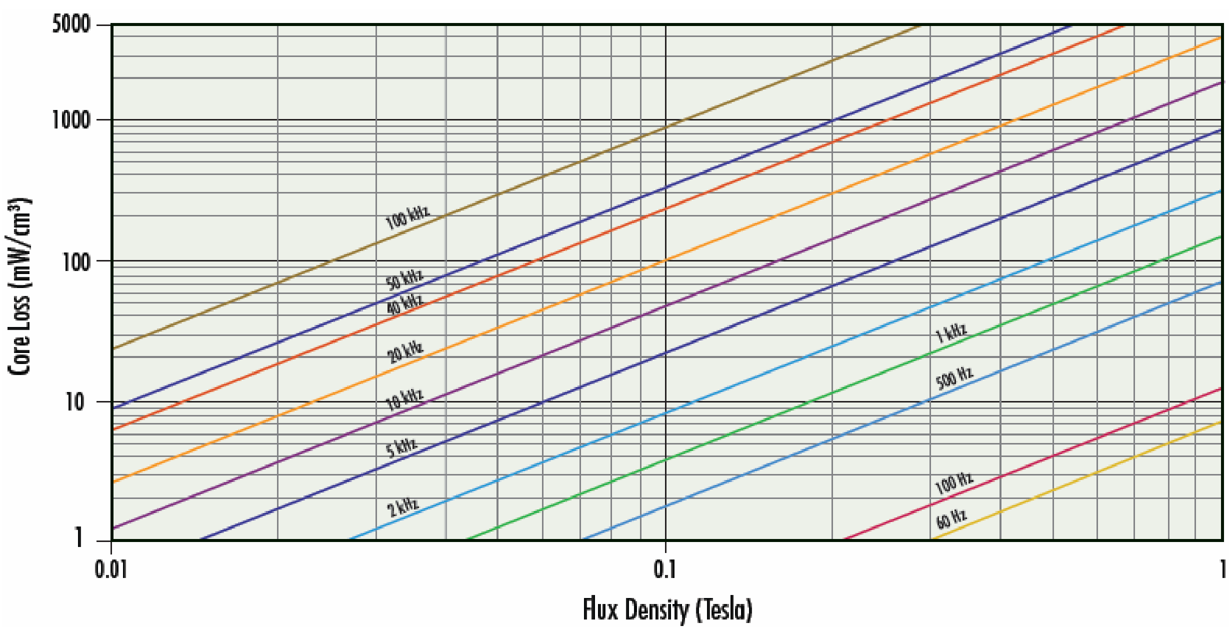
Core Type	Field inductance, $\mathcal{L}_0$, nH	Magnetic path length, l , mm	Magnetic path cross-sectional area, A, mm^2	Core magnetic volume, V, cm^3	Outside diameter, OD, mm	Inside diameter, ID, mm	Toroid height, h, mm
58381	43	41.4	23.2	0.960	18.1	9.01	7.12
59381	43	41.4	23.2	0.960	18	9.02	7.11

The goal is to find the energy densities, w_L, of the materials for the two same-size cores. To compare the energy densities, the maximum $\hat{B}_{\sim}$ and $\overline{H}$ are found, then ΔW_L is calculated for each.

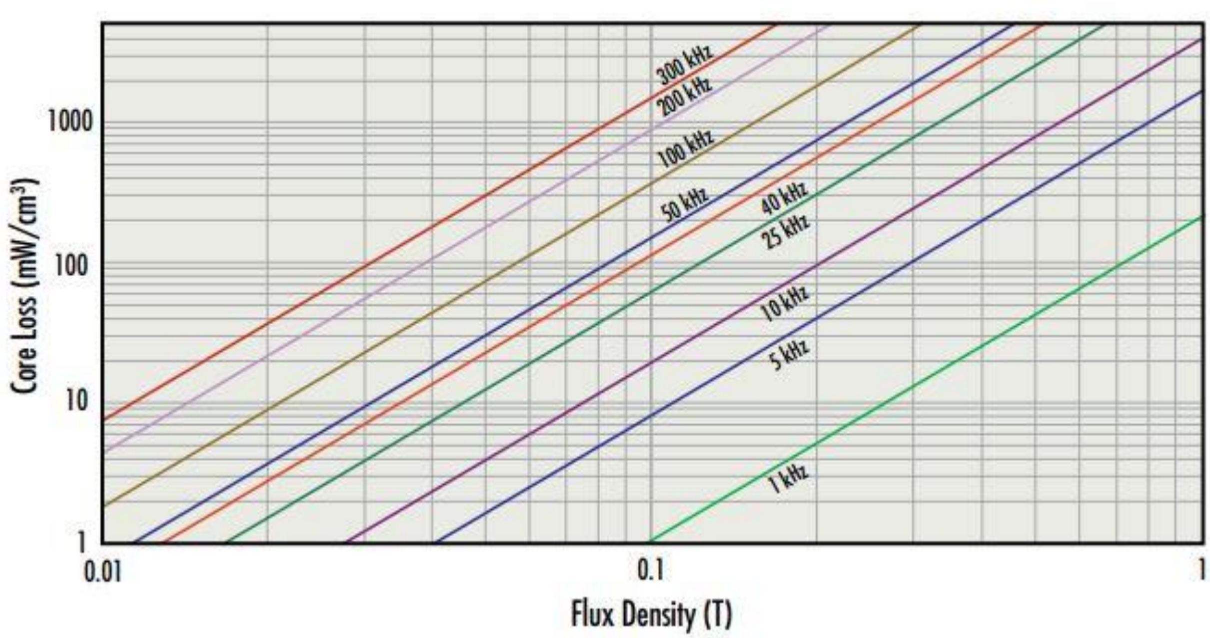
Power Loss and Δ*B*

First, the maximum $\hat{B}_{\sim}$ is derived from the loss curves or calculated from the loss equation. The power loss catalog curves for each are given below.

High Flux Power Loss Graph



Edge Power Loss Graph



The maximum $\hat{B}_{\sim}$, graphed on the horizontal axis, is determined by f_s, which selects one of the family of plots, and by $\overline{p}_c$. The value of $\overline{p}_c$ results from a core-size dependent thermal analysis. With a value of $\overline{p}_c$, and given the converter f_s, a $\hat{B}_{\sim}$ can be found from the graphs. (Note that f_s is the *core* and not necessarily the *circuit* switching frequency; push-pull circuits, for instance, switch the core at half the converter frequency.) For this comparison, with equal geometries and material of comparable thermal resistance, the same thermal analysis can be applied to both.

A simplified shape-based thermal analysis method is found in [1] that determines $\bar{p}_c$ from three design formulas:

$$r \approx 0.6204 \cdot V^{1/3} = \text{equivalent thermal radius of a (worst-case) sphere}$$

$$\bar{p}_c(\text{sphere}) = \frac{\Delta T}{R_\theta \cdot V} = \frac{\Delta T}{(8.33\,\text{cm}\cdot\text{K/W})\cdot r^2 + (167\,\text{cm}^2\cdot\text{K/W})\cdot r} = \bar{p}_c \text{ of an equivalent sphere of core material}$$

$$\bar{p}_c = \Xi_\theta \cdot (1 - f_w/2)\cdot \bar{p}_c(\text{sphere}) \quad \text{(preferably in mW/cm}^3\text{)}$$

where Ξ_θ is the *thermal shape factor*, a geometric factor of how much better the core shape is thermally than the worst-case sphere. For rectangular cross-section toroids, $\Xi_\theta = 1.63$; a rectangular toroid shape is this much better at "getting rid of heat" than a sphere. The final factor in $\bar{p}_c$ is $(1 - f_w/2)$. It accounts for the thermal effect of the core-winding configuration, from thermal conduction through the core of winding heat, where f_w is the fraction of winding heat that flows through the core. For toroids, it is approximately zero because the windings are outside from the core. This core-winding configuration factor is an approximate adjustment to the shape-based factors.

The final undetermined parameter is the maximum allowable ΔT. Ferrite $\bar{p}_c(T)$ curves typically reach a minimum around 90 °C. Above this temperature, the slope is positive and positive thermal feedback occurs. Powder cores exhibit relatively flat loss vs. temperature curves. Typical design targets do not let power components exceed 100 °C. Therefore, for a maximum ambient temperature of 50 °C and a choice of maximum core temperature of 90 °C, then ΔT = 40 °C = 40 K. Calculating from the above equations and data,

$$r = 0.612\text{ cm}\ ;\ \bar{p}_c(\text{sphere}) = \Delta 40\text{ K}/(3.12 + 102.2)\text{ cm}^3\cdot\text{K/W} = 380\text{ mW/cm}^3\ ;\ \bar{p}_c = 619\text{ mW/cm}^3$$

With this value of $\bar{p}_c$, the plot of 100 kHz on the above graphs gives the values:

$$\hat{B}_\sim \text{ (High Flux)} = 0.08\text{ T} = 80\text{ mT}$$

$$\hat{B}_\sim \text{ (Edge)} = 0.13\text{ T} = 130\text{ mT}$$

For $\hat{B}_\sim$, the Edge powder core has a significant 63% advantage over the High Flux powder core.

Saturation and Average *H*

The op-pt of a core at $\overline{H}$ corresponds to the average winding current, $\bar{i} = I$. For CCM current, this is the full-cycle average. For DCM current, it is the on-time average, given that the Δi during on-time is small relative to the average (the small-ripple assumption). Minimization of magnetizing current for transformers is desirable in converter design when it is not involved in primary-to-secondary winding power transfer. Magnetizing-current minimization is usually necessary anyway, to keep core power-loss within bounds, for current ripple is associated with it.

The maximum allowable value of $\overline{H}$ depends on how far into saturation the core can be driven. Ferrites are different from powder cores because they sustain μ (or $\mathcal{L}$ or circuit L) near the zero-current value of μ_i until at some relatively low value of $\overline{H}$, μ plunges quickly to near unity. For ferrite design, this $\mu(H)$ "cliff" is to be avoided under all operating conditions (including maximum temperature). For powder cores, we have more freedom because core saturation is soft and occurs over a very wide range. MnFe ferrites typically reach hard saturation over about a third of a decade, whereas powder cores retain useful μ for several decades.

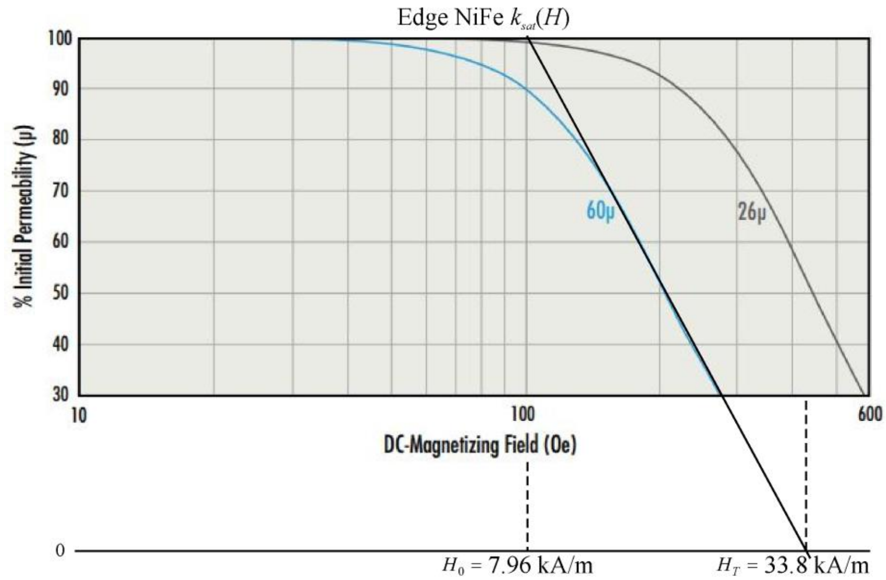
The "saturation region" of μ is approximately logarithmic and can be approximated with asymptotic line segments in the same way that Bode or frequency-response plots are simplified in dynamic circuit analysis. The left end of the region begins with a value of H_0 where rolloff begins, and ends at an asymptotic value of H_T. This approximate saturation model is shown below for the Edge NiFe 60μ material. Note that it is just a useful model – the actual material flattens and only approaches unity permeability asymptotically at very high applied current.

The parameter that quantifies saturation is the vertical-axis *fractional saturation*, k_{sat}, defined as

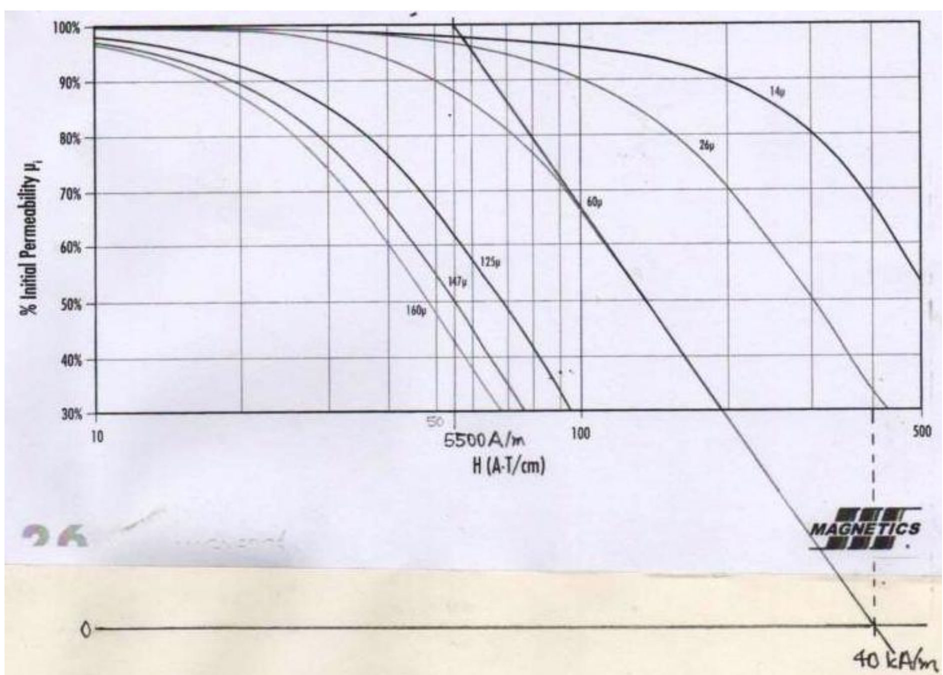
$$k_{sat} = \frac{\mu}{\mu_i} = \frac{\mu}{\mu(0\text{ A})} = \frac{\mathcal{L}}{\mathcal{L}_0} = \frac{L}{L(0\text{ A})}$$

where $k_{sat} = 1$ has no saturation and at $k_{sat} = 0$ is entirely saturated. The range of the saturation region on the semi-log graph is $\log(H_T/H_0)$ and for Edge 60μ cores, it is

$$\log(H_T/H_0) \approx \log(33.8/7.96) \approx 0.628$$



For High Flux 60 μ cores, it is found similarly from the material saturation graph, shown below as extrapolated to H_T: H_0 = 5.5 kA/m, H_T = 40 kA/m, $\log(H_T/H_0)$ = 0.862. The High Flux "saturation region" is wider than for Edge. More significantly, however, Edge has an "edge" over High Flux in that its "saturation region" begins at a higher H_0. Thus, Edge can be driven to a higher field current, $NI = N{\cdot}I$ than High Flux, where N is winding turns. And that will increase magnetic energy density, w_L.



Consequently, a maximum $\overline{H}$ depends on a minimum allowable k_{sat}. If the goal is to maximize inductance, L, then

$$L_{\max} = N_{\max}^2 \cdot k_{sat} \cdot \mathcal{L}_0 \ ; \ \ N_{\max} = \frac{H_T \cdot l}{I \cdot \sqrt{e}} \ ; \ k_{sat}(L_{\max}) = \frac{\log \sqrt{e}}{\log(H_T / H_0)} \approx \frac{0.2171}{\log(H_T / H_0)}$$

where N_{max} = number of turns at maximum L, L_{max}. Ordinarily for power applications, L_{max} is not the optimal op-pt because for typical materials, $k_{sat}(L_{max})$ is low – under 0.217 – and N_{max} is larger than what optimizes other magnetics performance factors. [2] Typically, for power conversion, k_{sat} is in a range from 0.7 to 0.5. Above 0.7, core energy density is underutilized and below 0.5 results in diminishing returns as the current waveform becomes increasingly superlinear, leading to control problems in the circuit design. Suppose circuit constraints result in a minimum allowable k_{sat} = 0.6. Then to continue the comparison of the two materials, values from the above graphs at k_{sat} = 0.6 are

$$\text{High Flux } \overline{H}(0.6) = 9.15 \text{ kA/m}$$

$$\text{Edge } \overline{H}(0.6) = 13.5 \text{ kA/m}$$

Thus, Edge has an advantage of 48% higher $\overline{H}(0.6)$ than High Flux.

With both ΔB and $\overline{H}(0.6)$ the energy density for the two core materials can be compared, remembering that a Tesla, $\text{T} \equiv \text{V·s/m}^2$:

$$\text{High Flux } w_L = [2\cdot(80 \text{ mT})]\cdot(9.15 \text{ kA/m}) = 1.46 \text{ kJ/m}^3 = 1.46 \text{ mJ/cm}^3$$

$$\text{Edge } w_L = [2\cdot(130 \text{ mT})]\cdot(13.5 \text{ kA/m}) = 3.51 \text{ kJ/m}^3 = 3.51 \text{ mJ/cm}^3$$

Then

$$\frac{\text{Edge } w_L}{\text{High Flux } w_L} = \frac{3.51 \text{ mJ/cm}^3}{1.46 \text{ mJ/cm}^3} = 2.4$$

Edge can store 2.4 times as much energy per volume of core as High Flux. At the same frequency, the core size is reduced for Edge by 2.4 times. For the example comparison, the two cores at 100 kHz have a transfer power of

$$\text{High Flux } \overline{P} = (1.46 \text{ mJ/cm}^3)\cdot(0.96 \text{ cm}^3)\cdot(100 \text{ kHz}) = 140 \text{ W}$$

$$\text{Edge } \overline{P} = (3.51 \text{ mJ/cm}^3)\cdot(0.96 \text{ cm}^3)\cdot(100 \text{ kHz}) = 336 \text{ W}$$

In conclusion, the newer Edge material has a significantly greater energy density than High Flux material, resulting in about a 34% decrease in linear dimensions for a magnetic component having an Edge core.

References

1. *Power Magnetics Design Optimization*, JUL19 revision, D. L. Feucht, Innovatia, innovatia.com
2. "How To Optimize Turns For Maximum Inductance With Core Saturation", APR 2019, how2power.com/pdf_view.php?url=/newsletters/1904/H2PowerToday1904_FocusOnMagnetics.pdf
3. www.how2power.com/pdf_view.php?url=/newsletters/1504/H2PowerToday1504_FocusOnMagnetics.pdf

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