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Technical Bulletin

Core Volume Minimization Theory

An important criterion in converter design is size, especially of magnetic components. This bulletin derives and explains how to use formulas for achieving the design goal of minimum core volume, V. The following derivation of design equations applies four constraints. Three are incremental or small-signal expressions of circuit flux change:

$$\Delta\lambda_p = L_p \cdot \Delta i_p = V_p \cdot (D \cdot T_s) = N_p \cdot \Delta\phi_p$$

where λ_p is the primary winding circuit flux of N_p primary turns, L_p is primary circuit inductance, ϕ_p is the field flux, and V_p is the voltage applied to the primary winding during on-time, $D{\cdot}T_s$. The primary winding handles the most power and places the greatest demand on core size. Then:

$$\Delta\phi_p = \Delta B \cdot A = (2 \cdot \hat{B}_{\sim}) \cdot A$$

where $\hat{B}_{\sim} = \Delta B/2$ is the amplitude of the B-field ripple and A is the core magnetic cross-sectional area. Another way to express $\Delta\phi_p$ is from:

$$\Delta\phi = \Delta B \cdot A = \mathcal{L} \cdot \Delta Ni$$

where $\mathcal{L}$ (A_L in catalog data) is the field inductance, L/N^2 and $Ni = N{\cdot}i$ = the field current. This equation is the flux-current relationship, $\lambda = L{\cdot}i$ for circuit flux, inductance, and current referred to the magnetic field of the core, as given in the following table. N is the referral parameter between corresponding field and circuit quantities.

Reference-Frame	**Current**	**Inductance**	**Flux**	**Voltage**
electrical circuit (terminal quantities)	circuit current, i	circuit inductance, L	circuit flux, $\lambda = N{\cdot}\phi$	circuit voltage, v
magnetic field	field current (MMF), $Ni = N{\cdot}i$	field inductance (per-turn-squared inductance, A_L), $\mathcal{L}$	field flux, ϕ	field voltage, v/N

Equating first and second flux-change expressions of the first equation above, and solving for the v-i relationship for the primary inductance,

$$L_p = \frac{V_p \cdot D \cdot T_s}{\Delta i_p}$$

Equating the circuit and field expressions for $\Delta\lambda_p$ (the second and third expressions) and solving for N_p,

$$N_p = \frac{V_p \cdot D \cdot T_s}{(2 \cdot \hat{B}_{\sim}) \cdot A}$$

This equation shows that N_p and $\hat{B}_{\sim}$ affect core size through A.

The previous two equations are related by circuit and field inductance:

$$L_p = N_p^2 \cdot \mathcal{L}$$

where the geometric formula for field inductance is

$$\mathcal{L} = \frac{\mu \cdot A}{l}$$

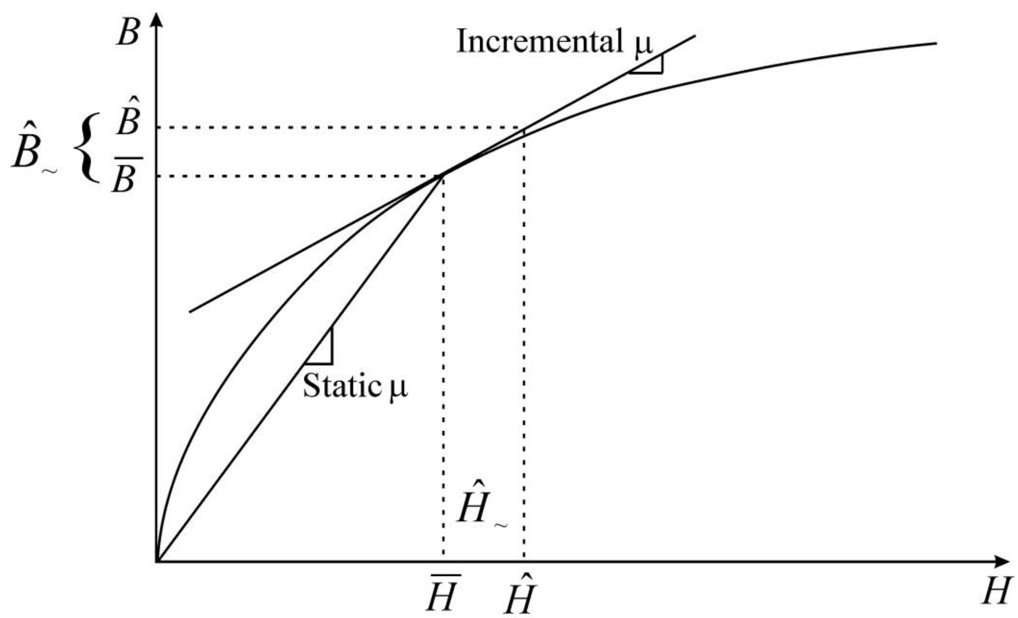
Then combining expressions for L_p and N_p,

$$\mathcal{L} = \frac{\mu \cdot A}{l} = \frac{L_p}{N_p^2} = \frac{\Delta\lambda_p / \Delta i_p}{(\Delta\lambda_p / \Delta\phi_p)^2} = \frac{(\Delta\phi_p)^2}{\Delta\lambda_p \cdot \Delta i_p} = \frac{(2 \cdot \hat{B}_{\sim})^2 \cdot A^2}{(V_p \cdot D \cdot T_s) \cdot \Delta i_p}$$

Solving the first and last expressions for core volume,

$$V = A \cdot l = \mu \cdot \frac{V_p \cdot D \cdot T_s \cdot \Delta i_p}{(2 \cdot \hat{B}_{\sim})^2} = \frac{(\Delta\lambda) \cdot (\Delta i_p)}{(\Delta B) \cdot (\Delta H)}$$

This relationship was derived from the $\Delta\lambda_p$ relationships, all of which apply incrementally around the *B*-*H* operating point (op-pt), $(\overline{H}, \overline{B})$, set by I_p. Δi_p and $\Delta B = 2 \cdot \hat{B}_{\sim}$ are incremental or small-signal variables that apply at the *B*-*H* op-pt, where μ is the incremental permeability, not the static μ_{static}, as shown on the graph below.



Permeability, μ, is a material property by which $\mathcal{L}$ varies with core size. Permeability is usually given in magnetics data as relative permeability, μ_r. Then

$$\mu = \mu_r \cdot \mu_0 = \mu_r \cdot (400 \cdot \pi \text{ nH/m}) \approx \mu_r \cdot 1.257 \text{ μH/m}$$

Permeability varies with *B*-*H* op-pt and for CCM converter operation should be taken as the incremental μ at the operating point, $\overline{B} = \hat{B} - \hat{B}_{\sim}$, where $\overline{B} = \mu_{static} \cdot \overline{H}$. If ΔB is small, then the variation of μ over the ΔB range can be regarded as negligible and μ considered constant.

The fourth constraint is the amount of static magnetic field that the core can support. This is a quiescent large-signal or total-variable quantity. The average magnetic core saturation is quantified by the op-pt magnetic field intensity, $\overline{H}$. By Ampere's Law,

$$\overline{H} \cdot l = N_p \cdot I_p \Rightarrow l = \frac{N_p \cdot I_p}{\overline{H}}$$

This saturation constraint leads to another expression for core volume by substituting for N_p from above:

$$V = A \cdot l = A \cdot \frac{N_p \cdot I_p}{\overline{H}} = N_p \cdot \frac{A \cdot I_p}{\overline{H}} = \left(\frac{V_p \cdot D \cdot T_s}{(2 \cdot \hat{B}_{\sim}) \cdot A} \right) \cdot \frac{A \cdot I_p}{\overline{H}} = \frac{V_p \cdot D \cdot T_s \cdot I_p}{(2 \cdot \hat{B}_{\sim}) \cdot \overline{H}} = \frac{(\Delta\lambda) \cdot I_p}{(\Delta B) \cdot \overline{H}}$$

This volume, unlike the previous expression, is derived from large-signal characteristics and contains op-pt parameter $\overline{H}$. Equating volumes results in

$$V = \frac{(\Delta\lambda) \cdot I_p}{(\Delta B) \cdot \overline{H}} = \frac{\Delta\lambda \cdot \Delta i_p}{\Delta B \cdot \Delta H}$$

The two expressions can be solved for the average *ripple factor*,

$$\gamma = \frac{\Delta i_p / 2}{I_p} = \frac{\Delta H / 2}{\overline{H}} = \frac{\hat{i}_{p\sim}}{\bar{i}_p} = \frac{\hat{H}_{\sim}}{\overline{H}}$$

which applies as much to field quantities as to circuit current waveforms. In its most basic and useful form,

$$V = \frac{P_p \cdot (D / f_s)}{\Delta B \cdot \overline{H}} = \frac{\overline{P}_p}{(\Delta B \cdot \overline{H}) \cdot f_s} = \frac{\text{circuit power}}{\text{field power density}}$$

This can be interpreted as a ratio of the per-cycle average primary winding power over magnetic power density. Under the condition that $\gamma = \gamma_{opt}$, for which $\Delta B \cdot \overline{H}$ is maximum, then V is minimized.

References

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2. “Utilizing Full Saturation and Power Loss To Maximize Power Transfer In Magnetic Components”, www.how2power.com/pdf_view.php?url=/newsletters/1504/H2PowerToday1102_FocusOnMagnetics.pdf

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